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Abstract: The term vector and elements of vector algebra are introduced in secondary education. In this section we recall the known facts about vectors and expand knowledge of algebra vectors and we will look at the basic properties of determinants of second and third order. Then these results will be used for applications in geometry, specifically the consideration of the main tasks of analytic geometry in area.

1. Field of random events

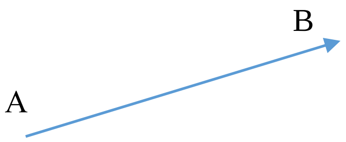
In mathematics and other sciences, working with various variables which are completely determined by measuring their number at selected single measures: the length of a segment, flat link, volume of body, time, temperature and more. They are scalar variables, but in physics, mechanics, ... there are sizes that are not fully determined only by measuring a number. Such are: strength, speed, acceleration and more. Each of these variables are completely determined that apart from the measuring number to know the direction. They are called vector or vector magnitudes or in short vectors.

2. Vectors and their coordinates

2.1. Targeted segments and the term vector

Each ordered pair of points A, B of the space [1] it's called line segment and it's marked as $\overline{AB}$. The A point is the start and B the end of the line segment $\overline{AB}$. If B coincides with A, then the pointed segment $\overline{AA}$, or $\overline{BB}$ we say that zero is pointed segment.

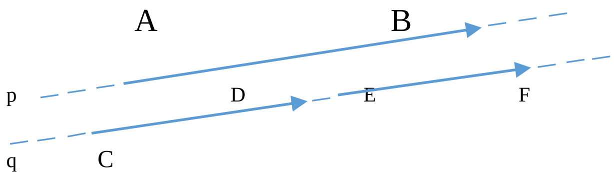
Each nonzero pointed segment $\overline{AB}$ graphically represents the arrow leading from the start A and end up in the end of B. This is shown in Picture 1.



Picture 1

For two not null pointed segments $\overline{AB}$ and $\overline{CD}$ we say that they are collinear if the lines AB and CD are parallel (where they match) it's $\overline{AB} \parallel \overline{CD}$.

The example in Picture 2 the lines p and q are parallel. Accordingly to that the pointed segments $\overline{AB}$, $\overline{CD}$ и $\overline{EF}$ are collinear. Each zero pointed segment will be considered collinear with each other pointed segment.



Picture 2

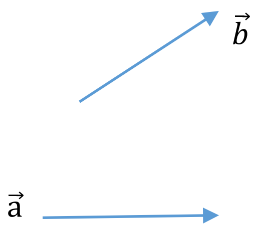
Pointed segments that equal each other are called **vector**. The vectors will be marked with small letters of the Latin alphabet a, b, ...

A vector a is determined by either its elements. So we can say that a class of all pointed segments equal to a pointed segments $\overrightarrow{AB}$ it is a vector representative $\overrightarrow{AB}$. We can write $\overrightarrow{AB} \in$ a.

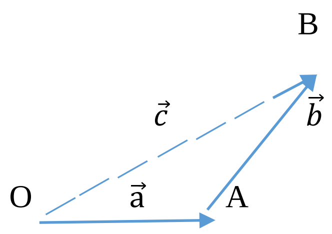
[1] The term area it means the same as in geometry, ordinary three-dimensional ("physical") area

1.2 Collecting vectors

The two given vectors a and b as shown in Picture 3. The vector a attached to a random point O and the vector b is attached of the last point A of a vector a, as shown in Figure 4. The last point of b by its binding to mark a with B. Then vector c = $\overrightarrow{OB}$ we call sum of the vectors a и b and we write c=a+b.

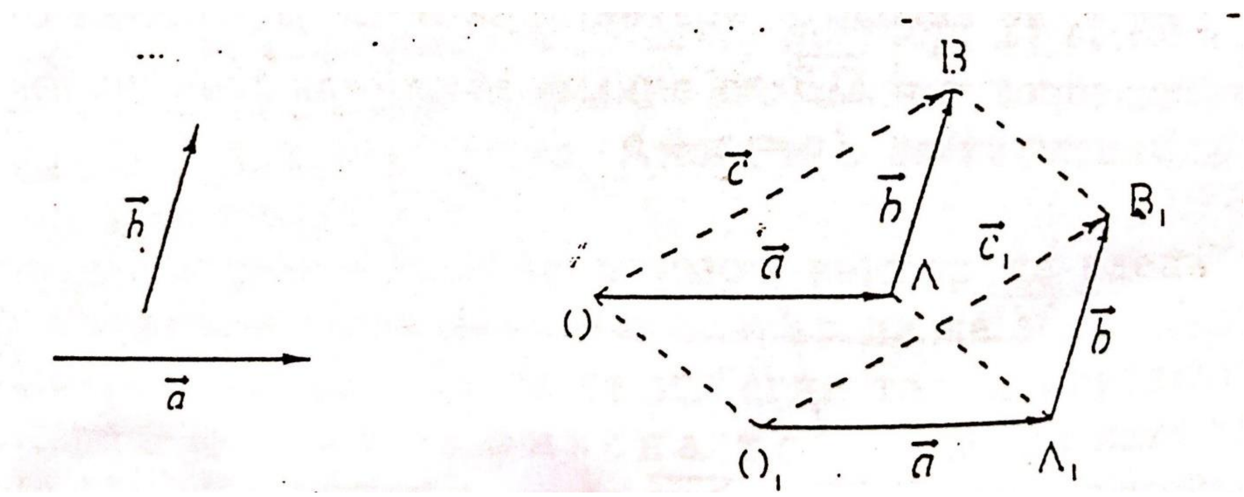


Picture 3



Picture 4

To be sure this operation defined collection of vectors is good should prove that the resulting vector c does not depend on the choice of specific representatives a and b. For this purpose besides the point O we can choose other arbitrary point O_1. Formore define will assume that a and b are not collinear. The point O_1 to connect it with a vector and the vector b - the last point A_1 of the vector a. The last point b be given the B_1 as shown in Picture 5. To mark the vector c1 whose representative is a targeted segment $\overrightarrow{O_1B_1}$.



Picture 5

Segments OA and O_1A_1 are parallel and of equal follows that quadrangle OO_1A_1A It is a parallelogram, and therefore the segments OO_1 and AA_1 are parallel and equal. For the same reasons we get that segments AA_1 and BB_1 are parallel and equal.

From these two conclusions it follows that the quadrilateral OO_1B_1B is a parallelogram of O_1B_1 and OB are parallel and equal, i.e $c_1 = \overrightarrow{O_1B_1} = \overrightarrow{OB} = c$.

The sum of c and b does not depend on the chosen representatives of a and b. Therefore the above applies only to the case when the vectors a and b are not collinear.

Let's note that the foregoing rule collection of vectors is known as the rule of triangular, but in this case when a and b are collinear vectors a, b and a + b does not form a triangle..

1.3. Multiplying of vector with real number

We define a vector multiplication operation number. Under the product of the real number λ vector and the mean vector λa so that:

(i) $|\lambda a| = |\lambda|\,|a|$

(ii) λa is collinear with a

(iii) for $\lambda > 0$ the vector has the same direction as, and $\lambda < 0$, λa has opposite direction from the direction of a.

Thus it is obtained that:

$1 \cdot a = a$ $(-1)\,a = -a$ $0 \cdot a = o = \lambda \cdot o$

for any vector and real number λ.